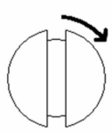
# Simple Harmonic Motion

# Oscillations

- Repetitive (periodic) motion
- Object moves around an equilibrium position



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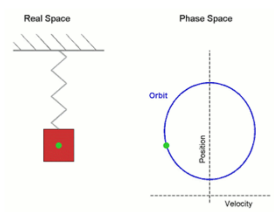


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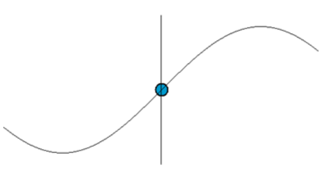
# Simple Harmonic Motion

- Periodic motion that is sinusoidal in nature



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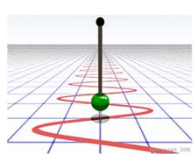

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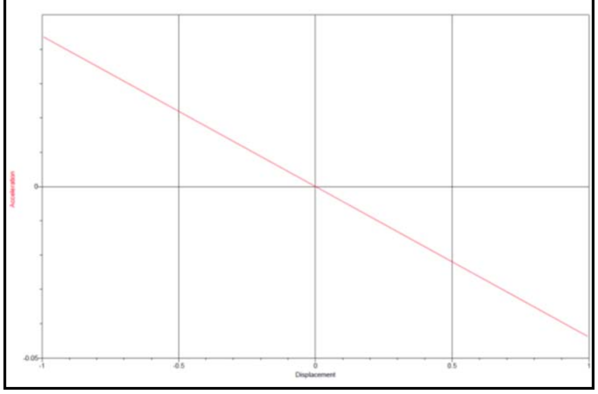
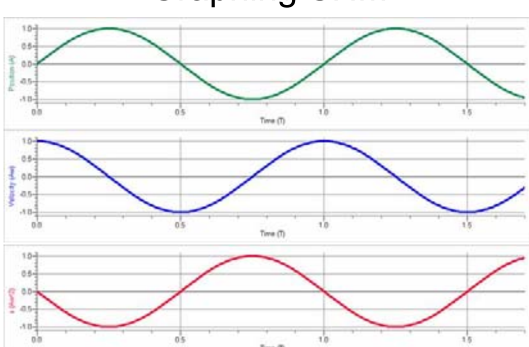
## Conditions Required for SHM

- The acceleration causing the motion is proportional to the displacement of the object from its equilibrium position
- The accelerating force must be trying to restore the object to its equilibrium position

$$a \propto -x$$



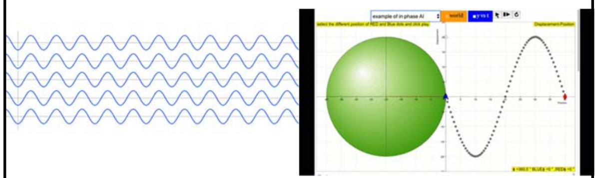

## Graphing SHM




farraguttn.com/science/milligan/apphys/shmover.htm

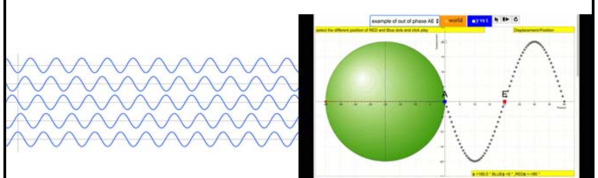
# Phase Difference

- When waves have the same frequency and starting point they are said to be in phase



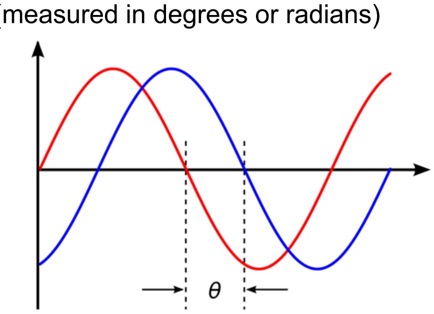


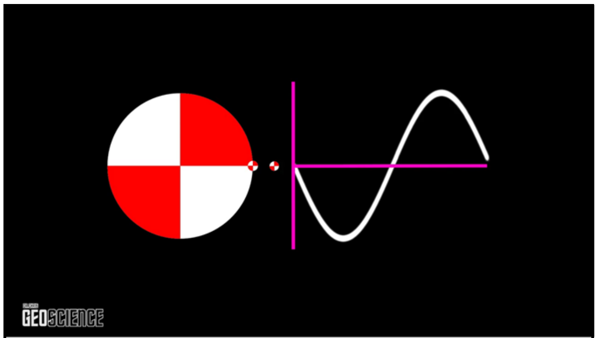
- When waves of the same frequency have different starting points they are said to be out of phase





- The difference between starting points between waves that are out of phase is the phase difference or phase shift (measured in degrees or radians)





Dr. Chris Geoscience
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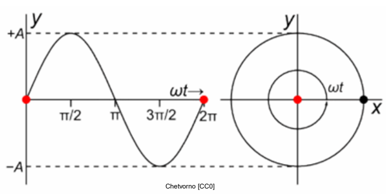
## SHM and Circular Motion

- SHM is related to circular motion
- An object moving in a circle with constant angular velocity has an angular displacement given by $\theta = \omega t$

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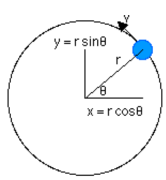


- The projection of the y component of the motion at a given time is $y=r\ sin\Theta$ or equivalently $y=r\ sin\ \omega t$

## Displacement, Velocity, & Acceleration

- Displacement can be represented by the equation:

$$\boxed{x = x_0 \sin \omega t}$$

- An equation for the velocity can be found by taking the derivative (calculus) of the displacement giving:

$$\boxed{v = \omega x_0 \cos \omega t}$$

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- The acceleration will be the derivative of the velocity:

$$a = -\omega^2 x_0 \sin \omega t$$

- But $x = x_0 \sin \omega t$

$$\boxed{a = -\omega^2 x}$$

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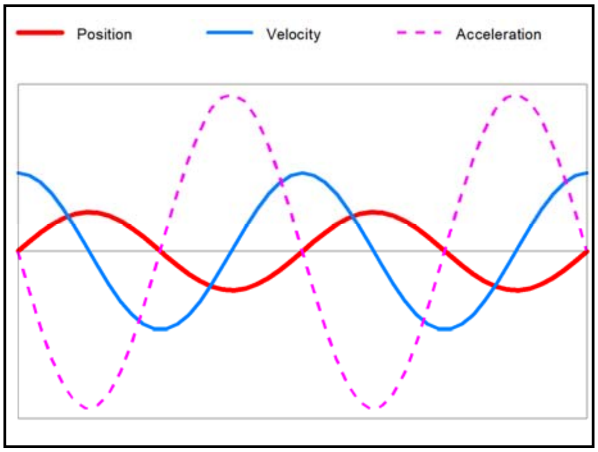


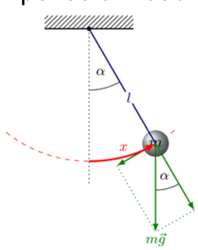
## The Velocity Equation

- We can derive an equation for the velocity of the particle at a given point

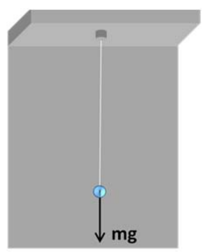
$$\boxed{v = \pm\omega\sqrt{(x_0^2 - x^2)}}$$

## Simple Pendulum

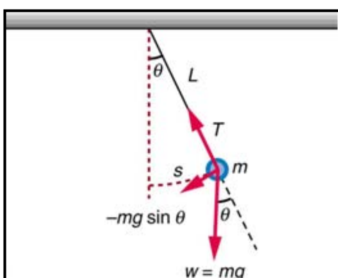
- The restoring force is perpendicular to the pendulum bob and string








https://opentextbc.ca/physicstestbook2/chapter/the-simple-pendulum/

$$-mg\sin\theta = ma$$

$$\sin\theta = \frac{x}{l}$$

$x$ – displacement in x direction
$l$ – length of pendulum

$$a = -\frac{gx}{l}$$

- This looks like the defining equation of SHM

$$a = -\left(\frac{g}{l}\right)x \qquad a = -\omega^2 x$$

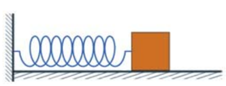
- Therefore… $\omega^2 = \frac{g}{l}$
- Since the period for SHM is given by $T = \frac{2\pi}{\omega}$

the period of the pendulum is

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

# Mass and Spring

- The restoring force for a mass on a spring is $F = -kx$



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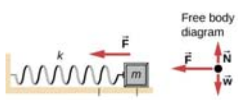


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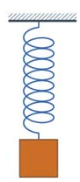

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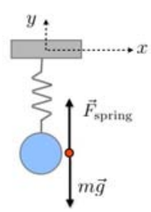



https://www.wired.com/2012/10/hackadays-portal-gun-and-magnetic-levitation/

$$-kx = ma$$

$$a = -\left(\frac{k}{m}\right)x$$

- Comparing this to the defining equation of SHM gives

$$\omega^2 = \frac{k}{m}$$

- And therefore a period of $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

# Energy

- A moving object has a kinetic energy of $E_k = \frac{1}{2}mv^2$
- The velocity of a particle in SHM at a given point is $v = \pm\omega\sqrt{x_0^2 - x^2}$
- Combining these equations gives us the kinetic energy at displacement $x$

$$\boxed{E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)}$$

- The maximum kinetic energy occurs when the displacement is zero

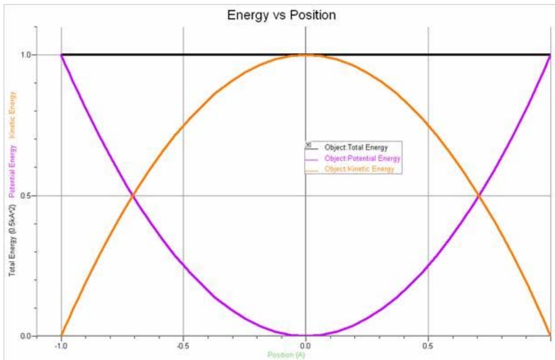
$$E_{k_{max}} = \frac{1}{2}m\omega^2 x_0^2$$

- This must be the total energy (as the potential energy at this point is zero)

$$\boxed{E_T = \frac{1}{2}m\omega^2 x_0^2}$$

- We can calculate the potential energy by subtracting kinetic energy from the total energy

$$E_P = E_T - E_k = \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

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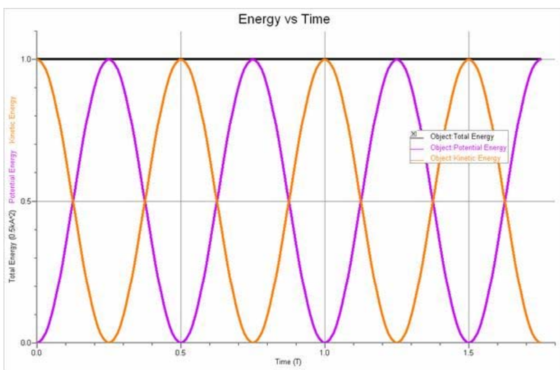


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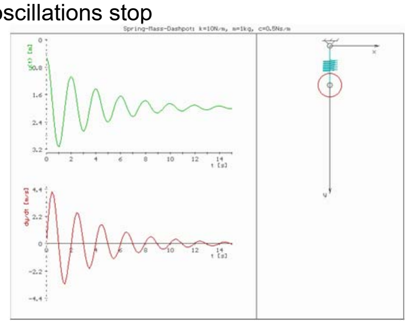
# Damping

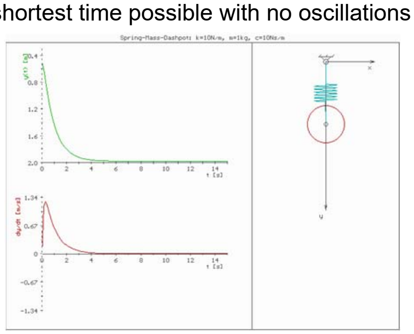
- All mechanical systems will vibrate when they are set in motion
- If a system is allowed to vibrate without any external forces being applied, it will vibrate at is natural frequency, $f_0$
- When resistive forces are present then the vibrations decay
- This is referred to as damping

- Underdamping
  - the amplitude gradually decreases until the oscillations stop



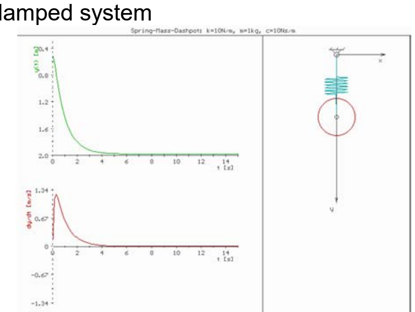


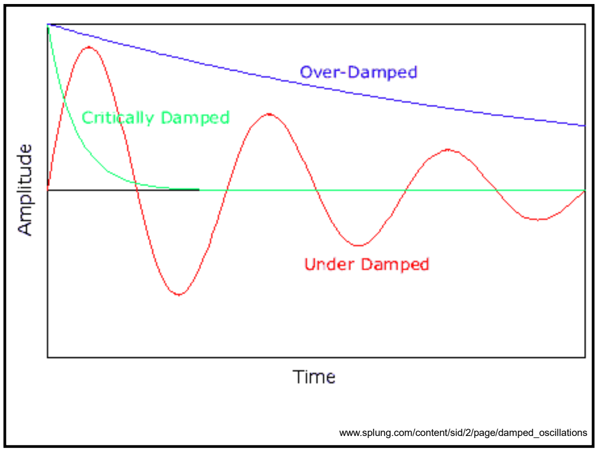

- Critical damping
  - The system returns to equilibrium in the shortest time possible with no oscillations






- Overdamping
  - The system returns to equilibrium with no oscillations but much slower than a critically damped system



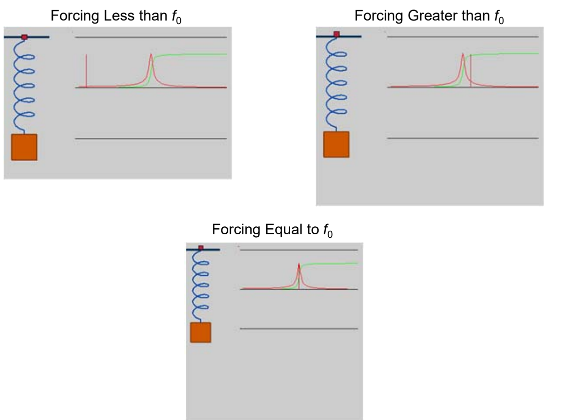



## Forced Vibrations

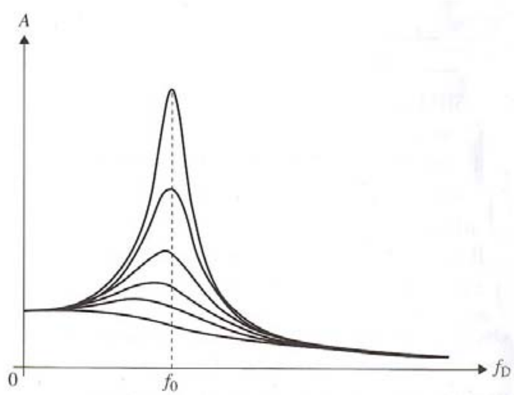
- When an external force acts on a mechanical system, the force may have its own frequency of vibration, which may affect the motion of the mechanical system
- Forced vibrations are those that occur when a regularly changing external force is applied to a system resulting in the system vibrating at the same frequency as the force

## Resonance

- When a mechanical system is forced to oscillate by a driving force that has the same frequency as the natural frequency ($f_0$) of the mechanical system, it will vibrate with maximum amplitude.
- This is called resonance.
- The degree of damping will alter the amplitude response of the system.






# Q factor

- The Q or “quality” factor is a criterion by which the sharpness of resonance can be assessed.

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$$

$$Q = 2\pi \times \text{resonant frequency} \times \frac{\text{energy stored}}{\text{power loss}}$$

- The Q factor is a numerical value with no units
- A system with a high Q factor is lightly damped
  - Energy dissipated per cycle is small
- The larger the Q factor, the sharper the resonance peak
- The Q factor is approximately the number of oscillations the system will make before its amplitude will decay to zero

- Some typical Q factors

| Oscillator | Q factor |
|---|---|
| critically damped door | 0.5 |
| mass on spring | 50 |
| simple pendulum | 200 |
| oscillating quartz crystal | 30 000 |

## Resonance can be bad…


AirMan454
PA-30 Twin Commanche Tail Flutter Test. NASA 1966

## Resonance can also be good…

- Microwave ovens
  - Microwaves force oscillation of water molecules generating heat inside the food
- Radios
  - The tuner uses resonance to select the station
- Quartz oscillators
  - The quartz is forced with an electric current causing it to oscillate at a very specific frequency